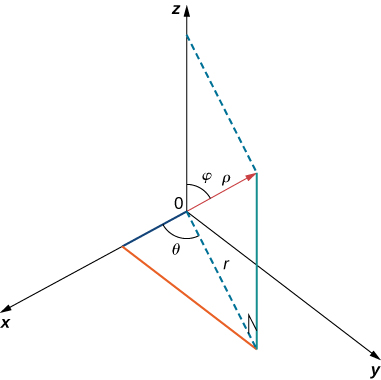
**Chapter 2**

**Vectors in Space**

**2.7 Cylindrical and Spherical Coordinates**

**Section Exercises**

**Use the following figure as an aid in identifying the relationship between the rectangular, cylindrical, and spherical coordinate systems.**



**For the following exercises, the cylindrical coordinates  of a pointare given. Find the rectangular coordinates  ofthe point.**

363. 

Answer: 

364. 

Answer: 

365. 

Answer: 

366. 

Answer: 

**For the following exercises, the rectangular coordinates  of a pointare given. Find the cylindrical coordinates  of the point.**

367. 

Answer: 

368. 

Answer: 

369. 

Answer: 

370. 

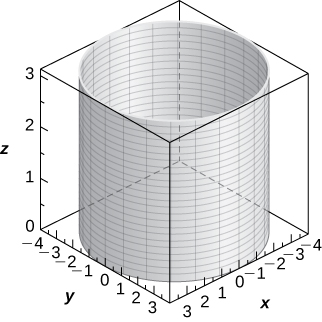
Answer: 

**For the following exercises, the equation of a surface in cylindrical coordinates is given.**

**Find the equation of the surface in rectangular coordinates. Identify and graph the surface.**

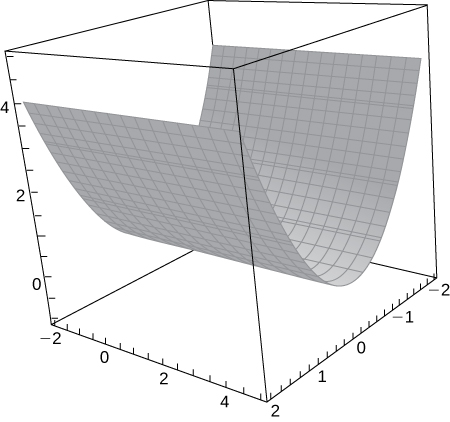
371. [T]

Answer: A cylinder of equation  with its center at the origin and rulings parallel to the axis,



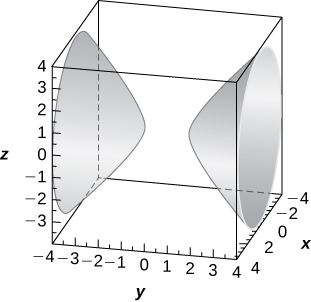
372. **[T]** 

Answer: A cylinder of equation  with rulings parallel to the axis,



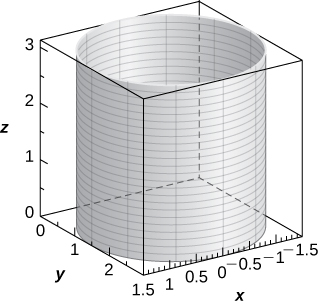
373. **[T]** 

Answer: Hyperboloid of two sheets of equation  with the axis as the axis of symmetry,



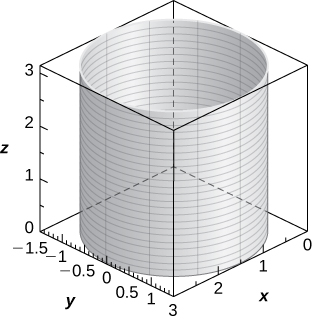
374. **[T]** 

Answer: Cylinder of equation  with a center at  and radius  with rulings parallel to the  axis,



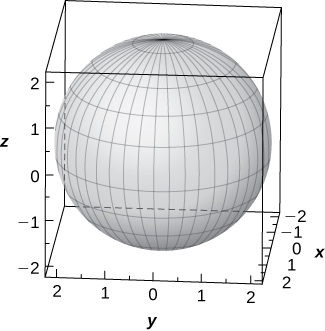
375. **[T]** 

Answer: Cylinder of equation  with a center at  and radius  with rulings parallel to the  axis,



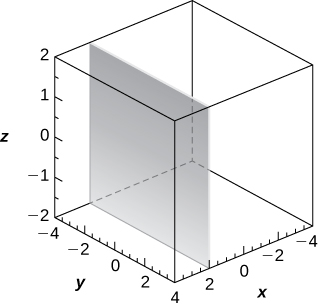
376. **[T]** 

Answer: Sphere of equation  centered at the origin of radius 



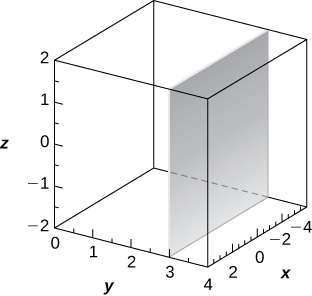
377. **[T]** 

Answer: Plane of equation 



378. **[T]** 

Answer: Plane of equation 



**For the following exercises, the equation of a surface in rectangular coordinates is given. Find the equation of the surface in cylindrical coordinates.**

379. 

Answer: 

380. 

Answer: 

381. 

Answer: 

382. 

Answer:  or 

383. 

Answer: 

384. 

Answer:  or 

**For the following exercises, the spherical coordinates  of a pointare given. Find the rectangular coordinates  of the point.**

385. 

Answer: 

386. 

Answer: 

387.

Answer: 

388. 

Answer: 

**For the following exercises, the rectangular coordinates  of a pointare given. Find the spherical coordinates  ofthe point. Express the measure of the angles in degrees rounded to the nearest integer.**

389. 

Answer: 

390. 

Answer: 

391. 

Answer: 

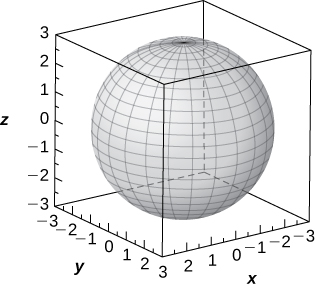
392. 

Answer: 

**For the following exercises, the equation of a surface in spherical coordinates is given. Find the equation of the surface in rectangular coordinates. Identify and graph the surface.**

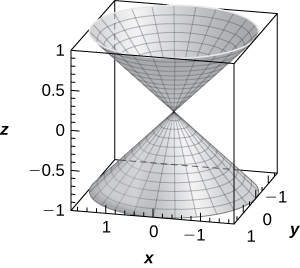
393. **[T]** 

Answer: Sphere of equation  centered at the origin with radius 



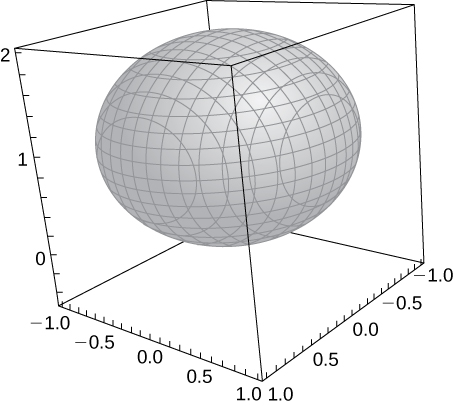
394. **[T]** 

Answer: Elliptic cone of equation  centered at the origin, with the axis as its axis of symmetry.



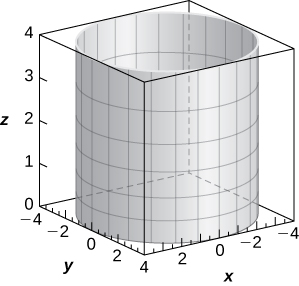
395. **[T]** 

Answer: Sphere of equation  centered at  with radius,



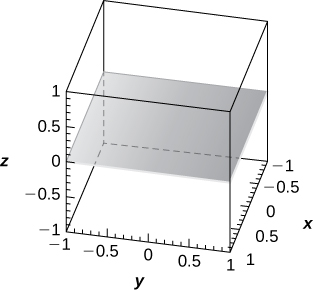
396. **[T]** 

Answer: Cylinder of equation  centered at the origin with radius  with rulings parallel to the axis,



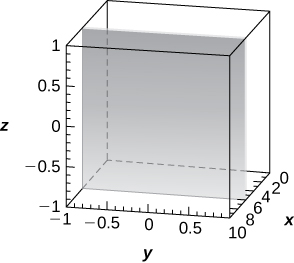
397. **[T]** 

Answer: The plane of equation 



398. **[T]** 

Answer: Plane of equation 



**For the following exercises, the equation of a surface in rectangular coordinates is given. Find the equation of the surface in spherical coordinates. Identify the surface.**

399.  

Answer:  or  Elliptic cone

400. 

Answer:  or  Sphere

401. 

Answer:  Plane at 

402. 

Answer:  Cylinder with radius 

**For the following exercises, the cylindrical coordinates of a pointare given. Find its associated spherical coordinates, with the measure of the angle  in radians rounded to four decimal places.**

403. **[T]** 

Answer: 

404. **[T]** 

Answer: 

405. 

Answer: 

406. 

Answer: 

**For the following exercises, the spherical coordinates of a pointare given. Find its associated** **cylindrical coordinates**.

407. 

Answer: 

408. 

Answer: 

409. 

Answer: 

410. 

Answer: 

**For the following exercises, find the most suitable system of coordinates to describe the solids.**

411. The solid situated in the first octant with a vertex at the origin and enclosed by a cube of edge length  where 

Answer: Cartesian system, 

412. A spherical shell determined by the region between two concentric spheres centered at the origin, of radii of  and  respectively, where 

Answer: Spherical system, 

413. A solid inside sphere  and outside cylinder 

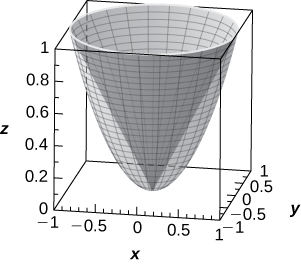
Answer: Cylindrical system, 

414. A cylindrical shell of height  determined by the region between two cylinders with the same center, parallel rulings, and radii of  and  respectively

Answer: Cylindrical system, 

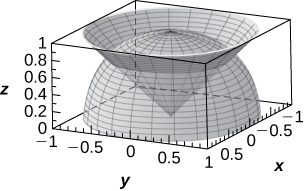
415. **[T]** Use a CAS to graph in cylindrical coordinates the region between elliptic paraboloid  and cone .

Answer: The region is described by the set of points .

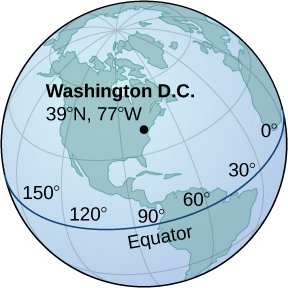


416. **[T]** Use a CAS to graph in spherical coordinates the “ice cream-cone region” situated above the *xy*-plane between sphere  and elliptical cone 

Answer: The region is described by the set of points 



417. Washington, DC, is located at  N and  W (see the following figure). Assume the radius of Earth is  mi. Express the location of Washington, DC, in spherical coordinates.



Answer: 

418. San Francisco is located at  and  Assume the radius of Earth is  mi. Express the location of San Francisco in spherical coordinates.

Answer: 

419. Find the latitude and longitude of Rio de Janeiro if its spherical coordinates are 

Answer:  

420. Find the latitude and longitude of Berlin if its spherical coordinates are 

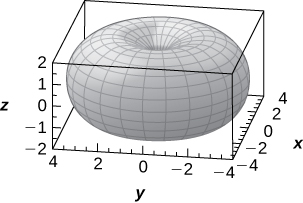
Answer:  

421. **[T]** Consider the torus of equation  where 

1. Write the equation of the torus in spherical coordinates.
2. If  the surface is called a *horn torus*. Show that the equation of a horn torus in spherical coordinates is 
3. Use a CAS to graph the horn torus with  in spherical coordinates.

Answer: a. , 

c.

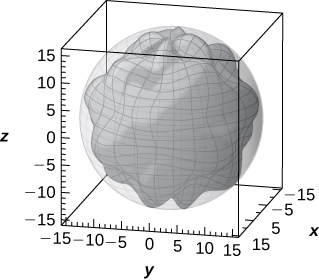


422. **[T]** The “bumpy sphere” with an equation in spherical coordinates is with  and  where  and  are positive numbers and  and  are positive integers, may be used in applied mathematics to model tumor growth.

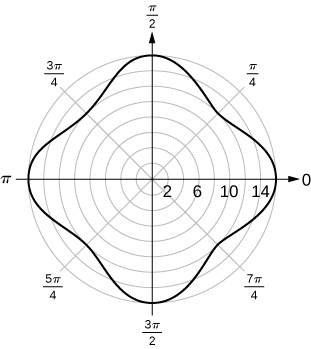
1. Show that the “bumpy sphere” is contained inside a sphere of equation  Find the values of  and  at which the two surfaces intersect.
2. Use a CAS to graph the surface for    and  along with sphere 
3. Find the equation of the intersection curve of the surface at b. with the cone  Graph the intersection curve in the plane of intersection.

Answer: a.  and 

b.



c.  where 



**Chapter Review Exercises**

**For the following exercises, determine whether the statement is true or false. Justify the answer with a proof or a counterexample.**

1. For vectors and and any given scalar  

Answer: True

1. For vectors and and any given scalar  

Answer: True

1. The symmetric equation for the line of intersection between two planes  and  is given by 

Answer: False

1. If  then  is perpendicular to 

Answer: True

**For the following exercises, use the given vectors to find the quantities**.

1. 
2. 
3. 
4. 
5. 

Answer: a. b.  c. Can’t dot a vector with a scalar; d. 

1. 
2. 
3. 
4. 
5. 
6. 

Answer: a.  b.  c.  d.  e. 

1. Find the values of  such that vectors  and  are orthogonal.

Answer:

**For the following exercises, find the unit vectors.**

1. Find the unit vector that has the same direction as vector  that begins at  and ends at 

Answer: 

1. Find the unit vector that has the same direction as vector  that begins at  and ends at 

Answer: 

**For the following exercises, find the area or volume of the given shapes.**

1. The parallelogram spanned by vectors 

Answer: 

1. The parallelepiped formed by  and 

Answer: 

**For the following exercises, find the vector and parametric equations of the line with the given properties.**

1. The line that passes through point  that is parallel to vector 

Answer: 

1. The line that passes through points  and 

Answer:

**For the following exercises, find the equation of the plane with the given properties.**

1. The plane that passes through point  and has normal vector 

Answer:

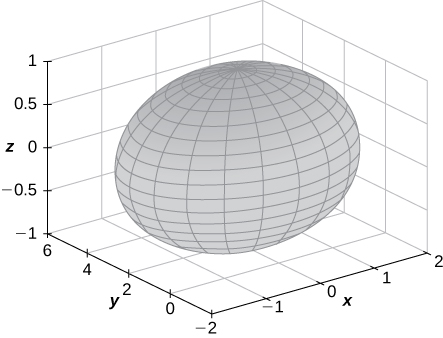
1. The plane that passes through points 

Answer:

**For the following exercises, find the traces for the surfaces in planes  Then, describe and draw the surfaces.**

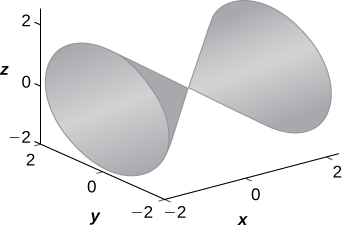
1. 

Answer: trace: is an ellipse,  trace:  is an ellipse,  trace:  is an ellipse. The surface is an ellipsoid.



1. 

Answer: trace:  is a circle,  trace:  is a hyperbola (or a pair of lines if   trace:  is a hyperbola (or a pair of lines if  The surface is a cone.



**For the following exercises, write the given equation in cylindrical coordinates and spherical coordinates.**

1. 

Answer: Cylindrical:  spherical: 

1. 

Answer: Cylindrical:  spherical: 

**For the following exercises, convert the given equations from cylindrical or spherical coordinates to rectangular coordinates. Identify the given surface.**

1. 

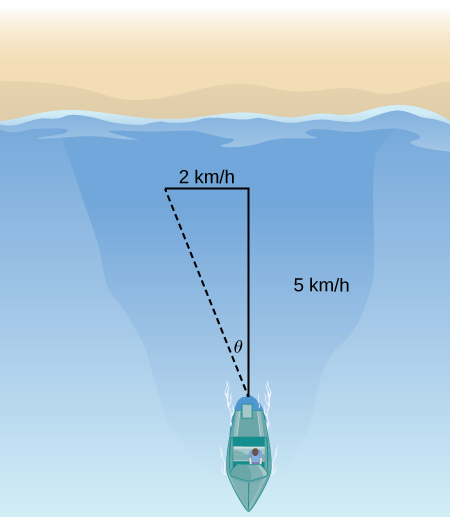
Answer: hyperboloid of one sheet

1. 

Answer:  sphere

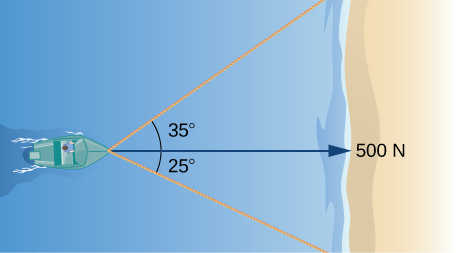
**For the following exercises, consider a small boat crossing a river.**

1. If the boat velocity is  km/h due north in still water and the water has a current of  km/h due west (see the following figure), what is the velocity of the boat relative to shore? What is the angle  that the boat is actually traveling?



Answer: Velocity:  km/h, angle : 

1. When the boat reaches the shore, two ropes are thrown to people to help pull the boat ashore. One rope is at an angle of  and the other is at  If the boat must be pulled straight and at a force of  find the magnitude of force for each rope (see the following figure).



Answer: 331 N, and 244 N

1. An airplane is flying in the direction of 52° east of north with a speed of 450 mph. A strong wind has a bearing 33° east of north with a speed of 50 mph. What is the resultant ground speed and bearing of the airplane?

Answer: Speed: mph, bearing: 

1. Calculate the work done by moving a particle from position  to  along a straight line with a force 

Answer: 

**The following problems consider your unsuccessful attempt to take the tire off your car using a wrench to loosen the bolts. Assume the wrench is  m long and you are able to apply a -N force.**

1. Because your tire is flat, you are only able to apply your force at a  angle. What is the torque at the center of the bolt? Assume this force is not enough to loosen the bolt.

Answer: J

1. Someone lends you a tire jack and you are now able to apply a -N force at an  angle. Is your resulting torque going to be more or less? What is the new resulting torque at the center of the bolt? Assume this force is not enough to loosen the bolt.

Answer: More,  J

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